

On the breaking of μ - τ flavor symmetry

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Abstract

In light of the observation of a relatively large θ_{13} , one has to consider breaking the μ - τ symmetry properly which would otherwise result in a vanishing θ_{13} (as well as $\theta_{23} = \pi/4$). Therefore, we investigate various symmetry-breaking patterns and accordingly identify those that are phenomenologically viable. Furthermore, the symmetry-breaking effects arising from some specific physics (e.g., the renormalization group equation running effect) are discussed as well [1].

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I. INTRODUCTION

As acknowledged by the 2015 Nobel prize in physics, the discovery of neutrino oscillations [2] implies that neutrinos are massive as opposed to the standard model (SM) setting. So far the most plausible way of generating small neutrino masses has been the seesaw mechanism [3] which leads neutrinos to be the Majorana particles. We therefore take on this possibility and deal with the symmetric neutrino mass matrix M_ν . Consequently, the neutrino mixing may arise from the mismatch between their mass and flavor eigenstates [4]. Such a mixing is described by one 3×3 unitary matrix $U = U_l^\dagger U_\nu$ with U_ν and U_l being respectively the unitary matrix for diagonalizing M_ν and the charged-lepton mass matrix $M_l M_l^\dagger$. In the standard parametrization, U is expressed as

$$U = P_\phi \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} P_\nu, \quad (1)$$

with the definition $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ (for $ij = 12, 13, 23$). The diagonal phase matrix $P_\nu = \text{Diag}(e^{i\rho}, e^{i\sigma}, 1)$ contains two Majorana CP phases, whereas $P_\phi = \text{Diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ consists of three unphysical phases that can be removed by the charged-lepton field redefinitions.

The mixing angles θ_{ij} as well as the Dirac CP phase δ can be measured in the neutrino-oscillation experiments [2]. It is interesting to find that θ_{23} has a value close to $\pi/4$ from the atmospheric neutrino-oscillation experiment [5]. In comparison, θ_{13} has not been determined until very recently [6] and was then only constrained by the upper limit $\sin^2 2\theta_{13} < 0.18$ [7]. Given these experimental facts, one was naturally tempted to take $\theta_{23} = \pi/4$ together with $\theta_{13} = 0$ as an ideal possibility which in turn motivated intensive studies about the μ - τ symmetry [8]. This symmetry is defined in a way that M_l is diagonal while M_ν keeps invariant under the transformation $\nu_\mu \leftrightarrow \nu_\tau$ and thus takes a form as

$$M_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix}. \quad (2)$$

It is straightforward to show that we will have $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ as a result of these symmetry conditions. Note that the μ - τ symmetry has no definite prediction for θ_{12} . Nevertheless, a further condition $A + B = C + D$ imposed on the M_ν given by Eq. (2) will give

$\sin \theta_{12} = 1/\sqrt{3}$ in which case we are left with the ever-popular tri-bimaximal (TB) mixing pattern [9].

However, the experimental results (i.e., the observed $\theta_{13} \simeq 0.15$ [6] as well as a possible deviation of θ_{23} from $\pi/4$) go against this simple flavor symmetry [10]:

$$\begin{aligned} \sin^2 \theta_{13} &= 0.0215 - 0.0259, & \Delta m_{21}^2 &= (7.32 - 7.80) \times 10^{-5} \text{eV}^2, \\ \sin^2 \theta_{23} &= 0.414 - 0.594, & |\Delta m_{32}^2| &\simeq |\Delta m_{31}^2| = (2.32 - 2.49) \times 10^{-3} \text{eV}^2. \end{aligned} \quad (3)$$

Here the results for neutrino mass squared differences $\Delta m_{ij}^2 = m_i^2 - m_j^2$ (for $ij = 21, 32, 31$) are also presented for later use. Note that the sign of Δm_{32}^2 (equivalently Δm_{31}^2) has not been determined yet, leaving us with two possibilities for the neutrino mass ordering (i.e., $m_1 < m_2 < m_3$ or $m_3 < m_1 < m_2$). In addition, the absolute neutrino mass scale remains unknown as well. Although the μ - τ symmetry must be broken to accommodate these experimental results [11], it may still be taken as a starting point for understanding the neutrino mixing pattern if in some situations this symmetry holds to a good approximation. In the next section we will study what kind of approximately μ - τ symmetric M_ν can lead to phenomenologically viable results. While section 3 is devoted to a further discussion about the symmetry-breaking effects induced by some specific physics (e.g., the renormalization group equation (RGE) running effects). Finally, we summarize our main results in the last section.

II. A GENERAL STUDY FOR THE SYMMETRY-BREAKING EFFECTS

Since the mixing matrix is derived from the mass matrix, we prefer to discuss the symmetry-breaking effects at the mass matrix level. In order to measure the symmetry-breaking strength, we introduce two dimensionless quantities [12]

$$\epsilon_1 = \frac{M_{e\mu} - M_{e\tau}}{M_{e\mu} + M_{e\tau}}, \quad \epsilon_2 = \frac{M_{\mu\mu} - M_{\tau\tau}}{M_{\mu\mu} + M_{\tau\tau}}, \quad (4)$$

which correspond to the defining features of μ - τ symmetry (i.e., $M_{e\mu} = M_{e\tau}$ and $M_{\mu\mu} = M_{\tau\tau}$). By virtue of these two quantities, the most general neutrino mass matrix can always

be parameterized into the form

$$M_\nu = \begin{pmatrix} A & B(1+\epsilon_1) & B(1-\epsilon_1) \\ B(1+\epsilon_1) & C(1+\epsilon_2) & D \\ B(1-\epsilon_1) & D & C(1-\epsilon_2) \end{pmatrix}. \quad (5)$$

When $|\epsilon_{1,2}|$ are simultaneously small enough (e.g., < 0.2), one can argue that M_ν assumes an approximate μ - τ symmetry. Instead of acquiring the possible values of $|\epsilon_{1,2}|$ via the reconstruction of M_ν in terms of the neutrino masses and mixing matrix U [13], we start from an approximately μ - τ symmetric M_ν (i.e., $|\epsilon_{1,2}|$ are assumed to be small in the first place) and explore its implications for θ_{13} and $\Delta\theta_{23}$. According to the naturalness argument, the sizes of θ_{13} and $\Delta\theta_{23}$ will be directly controlled by $\epsilon_{1,2}$. By making perturbation expansions for the small parameters in diagonalizing the M_ν given by Eq. (5), one will arrive at the following relations connecting θ_{13} and $\Delta\theta_{23} \equiv \theta_{23} - \pi/4$ to $\epsilon_{1,2}$ [12]:

$$\begin{aligned} \theta_{13}e^{-i\delta} &= (2\Delta m_{31}^2)^{-1}[2m_3m_{12}c_{12}^2\epsilon_1 + 2\bar{m}_1m_{12}^*c_{12}^2\epsilon_1^* + m_3(m_{22} + m_3)c_{12}s_{12}\epsilon_2 \\ &\quad + \bar{m}_1(m_{22}^* + m_3)c_{12}s_{12}\epsilon_2^*] + (2\Delta m_{32}^2)^{-1}[2m_3m_{12}s_{12}^2\epsilon_1 + 2\bar{m}_2m_{12}^*s_{12}^2\epsilon_1^* \\ &\quad - m_3(m_{22} + m_3)c_{12}s_{12}\epsilon_2 - \bar{m}_2(m_{22}^* + m_3)c_{12}s_{12}\epsilon_2^*], \\ \Delta\theta_{23} &= \text{Re}\{(2\Delta m_{31}^2)^{-1}[2m_{12}c_{12}s_{12}(\bar{m}_1^*\epsilon_1 + m_3\epsilon_1^*) + (m_{22} + m_3)s_{12}^2(\bar{m}_1^*\epsilon_2 + m_3\epsilon_2^*)] \\ &\quad - (2\Delta m_{32}^2)^{-1}[2m_{12}c_{12}s_{12}(\bar{m}_2^*\epsilon_1 + m_3\epsilon_1^*) - (m_{22} + m_3)c_{12}^2(\bar{m}_2^*\epsilon_2 + m_3\epsilon_2^*)]\}, \end{aligned} \quad (6)$$

where we have defined

$$m_{11} = \bar{m}_1c_{12}^2 + \bar{m}_2s_{12}^2, \quad m_{12} = (\bar{m}_1 - \bar{m}_2)c_{12}s_{12}, \quad m_{22} = \bar{m}_1s_{12}^2 + \bar{m}_2c_{12}^2, \quad (7)$$

with $\bar{m}_1 \equiv m_1e^{2i\rho}$ and $\bar{m}_2 \equiv m_2e^{2i\sigma}$. With the help of these results, one can study the dependence of $\theta_{13}e^{-i\delta}$ and $\Delta\theta_{23}$ on $\epsilon_{1,2}$ in some special situations to be given below.

First of all, let us work under the assumption of CP conservation in which case Eq. (6) is reduced to

$$\begin{aligned} \theta_{13} &= \frac{2m_{12}c_{12}^2\epsilon_1 + (m_{22} + m_3)c_{12}s_{12}\epsilon_2}{2(m_3 \mp m_1)} + \frac{2m_{12}s_{12}^2\epsilon_1 - (m_{22} + m_3)c_{12}s_{12}\epsilon_2}{2(m_3 \mp m_2)}, \\ \Delta\theta_{23} &= \frac{2m_{12}c_{12}s_{12}\epsilon_1 + (m_{22} + m_3)s_{12}^2\epsilon_2}{2(m_3 \mp m_1)} - \frac{2m_{12}c_{12}s_{12}\epsilon_1 - (m_{22} + m_3)c_{12}^2\epsilon_2}{2(m_3 \mp m_2)}, \end{aligned} \quad (8)$$

where \mp correspond to $\bar{m}_1 = \pm m_1$ (and $\bar{m}_2 = \pm m_2$). It is found that the values of θ_{13} and $\Delta\theta_{23}$ are strongly dependent on the neutrino mass spectrum as well as the Majorana phases

once the symmetry-breaking strength is specified. (1) When m_1 is vanishingly small, θ_{13} is well approximated by

$$\theta_{13} \sim \frac{1}{2} \sqrt{\frac{\Delta m_{21}^2}{\Delta m_{31}^2}} c_{12} s_{12} (2\epsilon_1 - \epsilon_2) \simeq 0.04 (2\epsilon_1 - \epsilon_2) , \quad (9)$$

which, given $|\epsilon_{1,2}| < 0.2$, is definitely unacceptable. (2) For $m_1 \simeq m_2 \gg m_3$, the results will depend on the combination of ρ and σ . When ρ is equal to σ , θ_{13} is extremely suppressed as shown by

$$\theta_{13} \sim \frac{1}{4} \frac{\Delta m_{21}^2}{\Delta m_{31}^2} c_{12} s_{12} (2\epsilon_1 - \epsilon_2) \simeq -0.004 (2\epsilon_1 - \epsilon_2) . \quad (10)$$

Otherwise, θ_{13} approximates to

$$\theta_{13} \sim \frac{1}{2} \cos 2\theta_{12} \sin 2\theta_{12} (2\epsilon_1 - \epsilon_2) \simeq 0.18 (2\epsilon_1 - \epsilon_2) , \quad (11)$$

which is still unable to give the observed value. (3) When neutrinos assume a nearly degenerate mass spectrum $m_1 \simeq m_2 \simeq m_3$ and $(\rho, \sigma) = (0, 0)$, one obtains θ_{13} as

$$\theta_{13} \sim \frac{2m_1^2}{\Delta m_{31}^2} \frac{\Delta m_{21}^2}{\Delta m_{31}^2} c_{12} s_{12} \epsilon_2 , \quad (12)$$

which is at most 0.03 by taking account the constraint $m_1 + m_2 + m_3 < 0.23$ eV from cosmological observations [14]. In the case of $(\rho, \sigma) = (0, \pi/2)$, one will have

$$\theta_{13} \sim \frac{2m_1^2}{\Delta m_{31}^2} c_{12} s_{12} (2c_{12}^2 \epsilon_1 + s_{12}^2 \epsilon_2) , \quad \Delta\theta_{23} \sim \frac{2m_1^2}{\Delta m_{31}^2} s_{12}^2 (2c_{12}^2 \epsilon_1 + s_{12}^2 \epsilon_2) . \quad (13)$$

Thanks to the enhancement factor $m_1^2/\Delta m_{31}^2$, θ_{13} can easily reach the observed value. Noteworthy, the correlation between θ_{13} and $\Delta\theta_{23}$ will give the prediction $|\Delta\theta_{23}| \sim \theta_{13} s_{12}/c_{12} \simeq 6^\circ$ which can be tested by precision measurements for θ_{23} . Finally, it turns out that the cases of $(\rho, \sigma) = (\pi/2, 0)$ and $(\pi/2, \pi/2)$ are not capable of generating a realistic θ_{13} or $\Delta\theta_{23}$.

When CP violation is concerned, more interesting possibilities will arise. In the first example we assume ρ and σ to be 0 or $\pi/2$ and $\epsilon_{1,2}$ to be purely imaginary (parameterized as $\epsilon_{1,2} = i|\epsilon_{1,2}|$). One immediately from Eq. (7) obtains $\Delta\theta_{23} = 0$, $\delta = \pm\pi/2$ and

$$\theta_{13} = \frac{2m_{12}c_{12}^2|\epsilon_1| + (m_{22} + m_3)c_{12}s_{12}|\epsilon_2|}{2(m_3 \pm m_1)} + \frac{2m_{12}s_{12}^2|\epsilon_1| - (m_{22} + m_3)c_{12}s_{12}|\epsilon_2|}{2(m_3 \pm m_2)} . \quad (14)$$

A similar analysis as in the CP conservation case shows that the observed θ_{13} is only obtainable under the condition of a nearly degenerate neutrino mass spectrum in combination

with $(\rho, \sigma) = (0, \pi/2)$ or $(\pi/2, 0)$. At this point it is worth mentioning that these results (i.e., trivial Majorana phases, maximal Dirac phase and $\theta_{23} = \pi/4$) are the same as those predicted by an M_ν respecting the μ - τ reflection symmetry [15]. Such an interesting symmetry is defined as follows: in the basis where M_l is diagonal, M_ν should keep invariant with respect to the transformation

$$\nu_e \rightarrow \nu_e^c, \quad \nu_\mu \rightarrow \nu_\tau^c, \quad \nu_\tau \rightarrow \nu_\mu^c, \quad (15)$$

and thus appears as

$$M_\nu = \begin{pmatrix} A & B & B^* \\ B & C & D \\ B^* & D & C^* \end{pmatrix}, \quad (16)$$

with A and D being real parameters. It is easy to check that the M_ν given by Eq. (5) happens to acquire this symmetry in the symmetry-breaking scenario under discussion. In another example, we instead assume ρ and σ to take non-trivial values and $\epsilon_{1,2}$ to be real. From Eq. (7) one can see that a finite δ may arise from the non-trivial Majorana phases even when $\epsilon_{1,2}$ themselves are real [16]. And its magnitude is not directly controlled by the symmetry-breaking parameters. This is easy to understand from that the μ - τ symmetry has no power of constraining the value of δ . For illustration, δ will be given by

$$\tan \delta = \frac{m_2 \sin 2\sigma - m_1 \sin 2\rho}{m_1 \cos 2\rho - m_2 \cos 2\sigma - m_3 \Delta m_{21}^2 / \Delta m_{31}^2}, \quad (17)$$

in the special case of $\epsilon_2 = 2\epsilon_1$ which as one can see later resembles the symmetry breaking induced by the RGE running effect.

So far the symmetry-breaking terms have been supposed to be relatively small as compared with the entry itself they reside in. But we will relax this constraint when dealing with an M_ν with a hierarchical structure. For an M_ν of this kind, one can assume that the dominant entries emerge at the leading order (LO) while the sub-dominant entries become finite only after receiving the next-to-leading-order (NLO) contributions which may also perturb the dominant entries. If the LO and NLO contributions are assumed to keep and break the μ - τ symmetry respectively, the sub-dominant entries will be completely occupied by the symmetry-breaking terms. This speculation motivates us to reconsider the physical implications of an approximately μ - τ symmetric M_ν . A good example in this regard is one

hierarchical neutrino mass matrix that will lead to $m_1 < m_2 \ll m_3$. It may be parameterized in a form as [17]

$$M_\nu = m \begin{pmatrix} d\epsilon & c\epsilon & b\epsilon \\ c\epsilon & 1+a\epsilon & -1 \\ b\epsilon & -1 & 1+\epsilon \end{pmatrix}, \quad (18)$$

where ϵ is a small quantity used to characterize the relative size of the NLO contributions compared to the LO ones, while a, b, c and d are $\mathcal{O}(1)$ real coefficients. This neutrino mass matrix leads us to the mixing angles

$$\theta_{13} \simeq \frac{1}{2\sqrt{2}} (b - c) \epsilon, \quad \Delta\theta_{23} \simeq \frac{1}{4} (a - 1) \epsilon, \quad (19)$$

and mass eigenvalues

$$m_{1,2} \simeq \frac{1}{4} \epsilon m (2d + a + 1 \mp \Delta), \quad m_3 \simeq 2m, \quad (20)$$

with $\Delta = \sqrt{(2d - a - 1)^2 + 8(b + c)^2}$. By fitting these mass eigenvalues with the measured Δm_{21}^2 and Δm_{32}^2 , one finds $\epsilon \sim \sqrt{\Delta m_{21}^2 / \Delta m_{32}^2} \simeq 0.15$. Hence the smallness of θ_{13} finds an explanation from the hierarchy between Δm_{21}^2 and Δm_{32}^2 in this particular scenario. Another example is the neutrino mass matrix

$$M_\nu = m \begin{pmatrix} e\epsilon & 1+d\epsilon & 1+c\epsilon \\ 1+d\epsilon & b\epsilon & \epsilon \\ 1+c\epsilon & \epsilon & a\epsilon \end{pmatrix}, \quad (21)$$

that results in $m_1 = -m_2$ and $m_3 = 0$ at the LO. Note that the LO terms respect the well-known $L_e - L_\mu - L_\tau$ symmetry [18] (with L standing for the lepton number) while the NLO terms violate it. A straightforward calculation yields the mixing angles

$$\theta_{13} \simeq \frac{(a - b) \epsilon}{2\sqrt{2}}, \quad \Delta\theta_{23} \simeq \frac{(c - d) \epsilon}{2}, \quad (22)$$

and mass eigenvalues

$$m_{1,2} \simeq \frac{1}{4} \left[(2e + 2 + a + b) \epsilon \mp 4\sqrt{2} \right] m, \quad m_3 \simeq \frac{1}{2} (a + b - 2) \epsilon m. \quad (23)$$

Fitting these mass eigenvalues with Δm_{21}^2 and Δm_{32}^2 requires ϵ to be at the order of $\Delta m_{21}^2 / \Delta m_{31}^2$, implying that θ_{13} would be exceedingly suppressed. Hence this pattern of M_ν is disfavored by the current experimental data.

III. SYMMETRY BREAKING ARISING FROM SOME SPECIFIC PHYSICS

In this section we study the symmetry-breaking effects arising from some specific physics. Above all, it should be noted that the RGE running effect may break the μ - τ symmetry. From the phenomenological point of view, flavor symmetries are usually implemented at a superhigh energy scale Λ_{FS} so as to keep away from the low-energy constraints. One should therefore take into account this effect when confronting the physical consequences of a flavor symmetry with the experimental data available at low energies Λ_{EW} [19]. In the RGE running process the significant difference between m_μ and m_τ will perform as a natural source for the symmetry breaking. In the minimal supersymmetry standard model (MSSM), the running of M_ν is governed by [20]

$$\frac{dM_\nu}{dt} = \left(Y_l^\dagger Y_l\right)^T M_\nu + M_\nu \left(Y_l^\dagger Y_l\right) + \alpha M_\nu, \quad (24)$$

where $\alpha \simeq -6/5g_1^2 - 6g_2^2 + 6y_t^2$ and $Y_l = \text{Diag}(y_e, y_\mu, y_\tau)$ denote the Yukawa couplings for charged leptons among which y_e and y_μ will be neglected in the following discussions. The reason for us to work in the MSSM is that the value of $y_\tau^2 = (1 + \tan^2 \beta)m_\tau^2/v^2$ may be greatly enhanced by choosing a large $\tan \beta$. A μ - τ symmetric M_ν at Λ_{FS} can be expressed in terms of the corresponding physical quantities in a form as

$$M_\nu = \begin{pmatrix} m_{11} & -\frac{1}{\sqrt{2}}m_{12} & -\frac{1}{\sqrt{2}}m_{12} \\ \cdots & \frac{1}{2}(m_{22} + m_3) & \frac{1}{2}(m_{22} - m_3) \\ \cdots & \cdots & \frac{1}{2}(m_{22} + m_3) \end{pmatrix}. \quad (25)$$

By integrating Eq. (24) one obtains the neutrino mass matrix at Λ_{EW} as [21]

$$\begin{aligned} M'_\nu &= I_\alpha \text{Diag}(1, 1, 1 - \Delta_\tau) M_\nu \text{Diag}(1, 1, 1 - \Delta_\tau) \\ &= I_\alpha \begin{pmatrix} m_{11} & -\frac{1}{\sqrt{2}}m'_{12}(1 + \frac{1}{2}\Delta_\tau) & -\frac{1}{\sqrt{2}}m'_{12}(1 - \frac{1}{2}\Delta_\tau) \\ \cdots & \frac{1}{2}(m'_{22} + m'_3)(1 + \Delta_\tau) & \frac{1}{2}(m'_{22} - m'_3) \\ \cdots & \cdots & \frac{1}{2}(m'_{22} + m'_3)(1 - \Delta_\tau) \end{pmatrix}, \end{aligned} \quad (26)$$

with

$$I_\alpha = \exp \left(\frac{1}{16\pi^2} \int_{\Lambda_{\text{FS}}}^{\Lambda_{\text{EW}}} \alpha dt \right), \quad \Delta_\tau = \frac{1}{16\pi^2} \int_{\Lambda_{\text{EW}}}^{\Lambda_{\text{FS}}} y_\tau^2 dt, \quad (27)$$

and

$$m'_{12} = m_{12}(1 - \frac{1}{2}\Delta_\tau), \quad m'_{22} = m_{22}(1 - \Delta_\tau), \quad m'_3 = m_3(1 - \Delta_\tau). \quad (28)$$

Numerically, for $\Lambda_{\text{FS}} = 10^{14}$ GeV, I_α and Δ_τ respectively range from 0.9 to 0.8 and from 0.002 to 0.044 when $\tan\beta$ varies from 10 to 50. The physical quantities at Λ_{EW} can be extracted by diagonalizing M'_ν with a unitary matrix U' . After a straightforward calculation one finds the mixing angles [22]

$$\begin{aligned} \theta'_{12} &\simeq \theta_{12} + \frac{1}{2}c_{12}s_{12}\frac{|\overline{m}_1 + \overline{m}_2|^2}{\Delta m_{21}^2}\Delta_\tau, & \theta'_{13} &\simeq c_{12}s_{12}\frac{m_3|\overline{m}_1 - \overline{m}_2|}{\Delta m_{31}^2}\Delta_\tau, \\ \theta'_{23} &\simeq \frac{\pi}{4} + \frac{|\overline{m}_1 + m_3|^2 s_{12}^2 + |\overline{m}_2 + m_3|^2 c_{12}^2}{2\Delta m_{31}^2}\Delta_\tau. \end{aligned} \quad (29)$$

From these results we can draw the following conclusions concerning the running behaviours of θ_{13} and θ_{23} : Even when the absolute neutrino mass scale reaches its upper limit from cosmological observations, $\tan\beta$ still should be larger than 50 in order to generate a realistic θ_{13} [23]. However, such a $\tan\beta$ would be problematic by rendering the bottom-quark Yukawa coupling non-perturbatively large [24]. It is thus fair to say that the observed θ_{13} can not be purely generated from the radiative effects [25]. As for θ_{23} , an appreciable deviation of it from $\pi/4$ can be acquired when the neutrino mass spectrum is nearly degenerate. Interestingly, this deviation will be positive (negative) in the case of $\Delta m_{31}^2 > 0$ (< 0), providing a potential correlation between the octant of θ_{23} and the neutrino mass ordering [26].

In the above discussions M_l has been taken to be diagonal. When this is not the case, the unitary matrix U_l will also contribute to the neutrino mixing according to $U = U_l^\dagger U_\nu$ [27]. Such a contribution may become relevant when a certain texture of M_ν fails to give viable phenomenological consequences or M_l is constrained to be non-diagonal by some physics (e.g., the connection between M_l with the mass matrix for down-type quarks in the grand unified theory (GUT) models). If U_ν results from a μ - τ symmetric M_ν , U_l may bring about the deviations of θ_{13} and θ_{23} from 0 and $\pi/4$. So let us explore the physical implications of μ - τ symmetry breaking from the charged lepton sector. To make things easier, a slightly different parametrization for the 3×3 unitary matrix from the standard one will be adopted:

$$U = U_{23}U_{13}U_{12}P_\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & \tilde{s}_{23}^* \\ 0 & -\tilde{s}_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{12} & \tilde{s}_{12}^* & 0 \\ -\tilde{s}_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & \tilde{s}_{13}^* \\ 0 & 1 & 0 \\ -\tilde{s}_{13} & 0 & c_{13} \end{pmatrix} P_\alpha, \quad (30)$$

with $\tilde{s}_{ij} = s_{ij}e^{i\delta_{ij}}$ (for $ij = 12, 13, 23$) and $P_\alpha = \text{Diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$. This new parametrization is related to the standard one via the phase transformations

$$\begin{aligned}\delta_{12} &= \phi_2 - \phi_1, & \delta_{13} &= \delta + \phi_3 - \phi_1, & \delta_{23} &= \phi_3 - \phi_2, \\ \alpha_1 &= \phi_1 + \rho, & \alpha_2 &= \phi_2 + \sigma, & \alpha_3 &= \phi_3.\end{aligned}\tag{31}$$

Correspondingly, the neutrino mixing will be obtained as

$$U = U_l^\dagger U_\nu = P_\alpha^\dagger U_{12}^{l\dagger} U_{13}^{l\dagger} U_{23}^{l\dagger} U_{23}^\nu U_{13}^\nu U_{12}^\nu P_\alpha^\nu = P_\alpha^{l\dagger} U_{23} U_{13} U_{12} P_\alpha^\nu.\tag{32}$$

Here we concentrate on the case of U_l being approximately diagonal for two considerations: the U_ν resulting from a μ - τ symmetric M_ν is already close to the realistic U , so the corrections from U_l need not be too significant; by analogy with the quark sector, an approximately diagonal U_l is expected as a natural outcome in light of the large mass hierarchies among charged leptons. As a result, the mixing angles in U approximate to [28]

$$\tilde{s}_{13} \simeq -\tilde{\theta}_{13}^l c_{23}^\nu - \tilde{\theta}_{12}^l \tilde{s}_{23}^\nu, \quad \tilde{s}_{12} \simeq \tilde{s}_{12}^\nu - \tilde{\theta}_{12}^l c_{12}^\nu c_{23}^\nu + \tilde{\theta}_{13}^l c_{12}^\nu \tilde{s}_{23}^{\nu*},\tag{33}$$

with $\theta_{23}^\nu = \pi/4$. These results can be further simplified by assuming $\theta_{13}^l \ll \theta_{12}^l$:

$$\delta \simeq \delta_{12}^l - \delta_{12}^\nu - \pi, \quad \theta_{13} \simeq \theta_{12}^l s_{23}^\nu, \quad s_{12} \simeq s_{12}^\nu + \theta_{13} c_{12}^\nu \cos \delta.\tag{34}$$

If θ_{12}^l has a value close to the Cabibbo angle of quark mixing $\theta_C \simeq 0.22$, then the second expression in Eq. (34) becomes $\theta_{13} \simeq \theta_C/\sqrt{2}$ which agrees well with the observations. This remarkable relation makes the idea of relating the lepton and quark sectors in the GUT models particularly attractive [29]. Moreover, the last expression in Eq. (34) implies a correlation between θ_{12}^ν and δ . For instance, if θ_{12}^ν has a value as in the TB mixing pattern (i.e., $\sin \theta_{12}^\nu = 1/\sqrt{3}$ which is close to the real θ_{12}), δ should lie around $\pm\pi/2$ so that the contribution from the second term to θ_{12} can be suppressed.

Finally, we point out that the mixing between active and sterile neutrinos can serve as another source for the symmetry breaking. Sterile neutrinos, as the name suggests, do not carry any quantum number under the SM gauge symmetry and thus do not take part in the SM interactions. Although there has not been direct evidence for sterile neutrinos, their existence is either theoretically motivated or experimentally hinted. A good example on the theoretical side is the heavy right-handed neutrino introduced for implementing the seesaw mechanism. On the experimental side, the long-standing LSND anomaly [30] and several

other short-baseline neutrino-oscillation anomalies [31] imply the possible existence of an $\mathcal{O}(\text{eV})$ sterile neutrino which mixes with the active neutrinos. It is therefore worthwhile for us to investigate the implementation of μ - τ symmetry in the presence of sterile neutrinos. One interesting possibility in this connection is just that sterile neutrinos may be responsible for the symmetry breaking [32]. To be specific, in the 3+1 neutrino mixing scheme (i.e., three active neutrinos plus one sterile neutrino), the 4×4 neutrino mass matrix can be parameterized as

$$M = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\mu} & m_{es} \\ m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} & m_{\mu s} \\ m_{e\mu} & m_{\mu\tau} & m_{\mu\mu} & m_{\tau s} \\ m_{es} & m_{\mu s} & m_{\tau s} & m_{ss} \end{pmatrix}. \quad (35)$$

Note that the upper-left 3×3 sub-matrix has been assumed to keep the μ - τ symmetry, while $m_{\mu s} \neq m_{\tau s}$ will be taken as the source for symmetry breaking. This would be a reasonable assumption when sterile neutrinos have a different mass origin from the active neutrinos so that the formers do not necessarily respect the symmetry possessed by the latters.

IV. SUMMARY

In summary, we have performed a systematic study on the various μ - τ symmetry breaking patterns in order to accommodate the observed θ_{13} in a proper way. In the first approach, two parameters $\epsilon_{1,2}$ are introduced to characterize the symmetry breaking and required to be small (e.g., $|\epsilon_{1,2}| < 0.2$) so as to keep the symmetry as an approximate one. When CP is conserved, an approximately μ - τ symmetric M_ν is capable of producing a viable θ_{13} only under the condition of a nearly degenerate neutrino mass spectrum in combination with $(\rho, \sigma) = (0, \pi/2)$ in which case a $|\Delta\theta_{23}| \simeq 6^\circ$ is also predicted. In the particular case that $\epsilon_{1,2}$ are purely imaginary while ρ and σ take trivial values, one is led to $\Delta\theta_{23} = 0$ and $\delta = \pm\pi/2$ as predicted by the μ - τ reflection symmetry. When an M_ν with a hierarchical structure is concerned, another approach may be invoked: the LO effects which respect the symmetry only contribute to the dominant entries, while the symmetry-breaking NLO effects are responsible for generating the sub-dominant entries as well as perturbing the dominant ones. One hierarchical M_ν that leads to $m_1 < m_2 \ll m_3$ turns out to be a good illustration for this approach of implementing the approximate μ - τ symmetry.

On the other hand, some specific physics that may give rise to the μ - τ symmetry breaking have been discussed as well. First of all, the RGE running effect always serves as a source for the symmetry breaking when this symmetry is implemented at an energy scale much higher than Λ_{EW} . However, this effect is not sufficient for generating the observed θ_{13} from 0 even in the optimal situation where the absolute neutrino mass scale and $\tan\beta$ take their largest allowed values. Furthermore, when M_l is not diagonal for some reasons, U_l will also contribute to the neutrino mixing. If U_l features $\theta_{12}^l \simeq \theta_C \gg \theta_{13}^l$ and U_ν results from an M_ν respecting the μ - τ symmetry, then an interesting relation $\theta_{13} \simeq \theta_C/\sqrt{2}$ is reached which has strengthened the motivation for relating the quark and lepton sectors. Last but not least, the mixing between active and sterile neutrinos (whose existence is hinted by a few short-baseline neutrino-oscillation anomalies) may also be responsible for the breaking of this interesting symmetry.

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